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RADIATIVE HEAT-TRANSFER ANALYSIS USING AN EFFECTIVE ABSORPTIVITY FOR ABSORPTION, EMISSION AND SCATTERING

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Abstract—In this study, a non-conventional approach is used for the solution of radiative heat-transfer equations through absorbing, emitting and scattering media. An expression has been developed herein, for the effective absorptivity of a layer of the medium which includes the effect of absorption, emission and scattering. This new "effective absorptivity in the presence of scattering" can be used to evaluate the radiative heat transfer between two surfaces with an intervening flowing or stationary medium. In this study, this expression is used in the evaluation of radiative heat transfer in turbulent boundary layer flows over a flat plate. The radiative heat-transfer results calculated by the method developed in this study are compared with the results calculated by conventional methods. It is seen that the method, developed herein, is accurate as well as faster than present techniques and thus saves considerable computer time.

NOMENCLATURE

a_n, a tabulated coefficient dependent on the particle size parameter and refractive index;
A, defined in equation (11);
B, defined in equation (12);
D, diameter of spherical particles;
e_b, black body emissive power;
E_n, Nth order exponential integral function;
G, defined in equation (A.2);
H, enthalpy;
I, radiation intensity;
K, coefficient defined by $K^2 = \alpha(\alpha+2\beta)$;
m, coefficient defined in equation (3);
$\bar{m}$, real part of the complex refractive index $\bar{m}(1-ix)$;
P, pressure;
Pr_{eff}, effective Prandtl number;
Pn, Nth order Legendre polynomial;
Q_R, radiative heat flux;
R, volumetric density of spontaneous radiation;
S, scattering function;
u, x-component of velocity;
v, y-component of velocity;
z, particle size parameter.

Greek symbols

α, absorption coefficient;
β, extinction coefficient;
γ, scattering coefficient;
δ, coefficient defined in equation (4);
μ, cosine of the angle between the direction of propagation and the x-axis;
μ_{eff}, effective viscosity;
ω, solid angle;
ρ, density;
τ, optical coordinate;
τ_0, optical thickness.

Subscripts

λ, monochromatic or wavelength dependent property;
x, pertaining to location x;
w, pertaining to wall;
∞, pertaining to freestream.

INTRODUCTION

A NUMBER of engineering problems involve the prediction of the radiative heat transfer through a medium which emits, absorbs, and scatters. Some of the important examples are radiative energy transfer through the medium of the boundary layer of a high speed vehicle (containing particles of ablative material), luminous flames, pulverized coal flames and rocket exhausts.

Chandrasekhar [1] gives a comprehensive discussion of the formulation of the equation of radiative heat transfer. Love [2] has also given a good discussion on handling the problem of scattering by suspended particles. One thing is evident; the presence of scattering in a medium considerably complicates the already complicated problem of radiative transfer. Many investigators, therefore, have neglected scattering in order to simplify the analysis. However, some investigators have included the effect of scattering in addition to absorption and emission. Notable among them is the work of Love and his associates [2–4]. Others who have done theoretical work of considerable importance are Beach *et al.* [5], Boles *et al.* [6], and Shahrokhi and Wolf [7]. Sanders and Lenoir [8] and Nagy and Lenoir [9] studied experimentally the radiative transfer through particles of aluminum oxide

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and graphite suspended in carbon tetrachloride. Hottel *et al.* [10] studied the bidirectional reflectance and transmittance of monodispersed polystyrene spheres and correlated the results with theory.

An excellent review of the differential methods of radiative transfer has been reported by Adrianov [11]. An analytical method for determination of the angular emission coefficients has been suggested by Surinov and Kobyshev [12]. Some other works reported in the Soviet literature on the radiative heat transfer are Kaganer [13], Adrianov [14], and Blokh [15]. Adrianov [14] suggested a method in which the effect of scattering and absorption by the particles suspended in an absorbing, emitting and scattering medium can be represented by a function called the effective absorptivity of a gas layer in the presence of scattering. In that paper, an example of radiation between two parallel plates at constant temperature was considered based on the following assumptions:

1. The medium intervening between the two surfaces absorbs and scatters but does not emit.
2. The medium is isothermal.
3. Characteristic scattering curves are axisymmetrical.

In the present study, the method of Adrianov has been developed further to include emission in the medium in addition to absorption and scattering. The method has been extended to account for variable temperature in the medium using a finite difference technique and a new effective absorptivity is defined and developed. This new absorptivity is given the name "effective absorptivity in the presence of scattering". Once this new effective absorptivity of the medium, which includes the effect of absorption, scattering and emission, is found, the problem of radiative exchange between the two surfaces is simplified. This method is applicable to the problem of radiative heat transfer between two surfaces with an intervening flowing or stationary medium. An expression for this effective absorptivity for the absorbing, emitting and scattering medium between two surfaces has been developed herein. This expression is used, in this study, in the evaluation of radiative heat transfer in turbulent boundary-layer flows over a flat plate, the equations for which are given in the Appendix. The boundary layer is divided into sublayers and effective absorptivity for each layer is calculated separately, and this is used to calculate the radiative heat transfer in the layer. For the solution of the boundary-layer equations, the Patankar and Spalding [21] technique, a finite-difference technique for the solution of the turbulent boundary layer equations, is used.

The radiative heat-transfer results calculated by the method developed in this study are compared with the results calculated by a conventional method, the equations for which are given in the Appendix. While there is not much difference in the calculations of the radiative heat transfer, the present method performs those calculations in considerably less time. It is seen that the saving in computer time is as much as 15% when the calculations are carried out up to a distance of 1.0 ft from the leading edge of the flat plate. Furthermore, the method can be incorporated easily into analysis techniques [17–19], where scattering has previously been neglected to avoid complicated analysis. Therefore, this method should prove to be very useful to investigators for including scattering by suspended particles into their analysis.

THEORETICAL DEVELOPMENT

Adrianov [4] showed that in a case of radiative heat transfer between two walls at constant temperatures and with an isothermal medium of absorbing–scattering gas, the problem is simplified by finding an absorptivity of the layer which takes into account the effect of scattering. In this study, this method has been extended to include a more general case of radiative heat transfer. The first generalization is that the gas is assumed to be emitting in addition to absorbing and scattering. Secondly, it is felt that there is no need to assume the gas to be at constant temperature throughout its section.

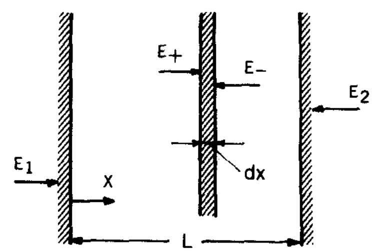

FIG. 1.

Consider a case of radiative heat transfer between two infinite parallel plates with an intervening medium of an absorbing, emitting and scattering gas. Consider a thin layer of the gas in the medium. Assume the radiant heat flux vector to be in the form of a difference of two fluxes in the X-direction flowing head on into one another.

The differential equations for the head on fluxes E_{λ_+} and E_{λ_-} can be written as (two flux approximation)

$$\frac{dE_{\lambda_+}}{dx} = -(\alpha_\lambda + \delta_{\lambda_+}\gamma_\lambda)m_{\lambda_+}E_{\lambda_+} + \delta_{\lambda_-}\gamma_\lambda m_{\lambda_-}E_{\lambda_-} + \frac{R_\lambda}{2} \quad (1a)$$

and

$$\frac{dE_{\lambda_-}}{dx} = (\alpha_\lambda + \delta_{\lambda_-}\gamma_\lambda)m_{\lambda_-}E_{\lambda_-} - \delta_{\lambda_+}\gamma_\lambda m_{\lambda_+}E_{\lambda_+} + \frac{R_\lambda}{2}. \quad (1b)$$

Here the subscripts $(+)$ and $(-)$ denote the positive and negative directions along the x-axis (Fig. 1) and

$$E_{\lambda_+} = \int_{2\pi_+} I_\lambda \mu \, d\omega \quad (2a)$$

$$E_{\lambda_-} = \int_{2\pi_-} I_\lambda \mu \, d\omega \quad (2b)$$

R_λ = Spectral volumetric density of spontaneous radiation

$= 4\alpha_\lambda e_{b\lambda}$

and

$$m_{\lambda_+} = \frac{\int_{2\pi_+} I_\lambda(\mu, x)\,d\omega}{\int_{2\pi_+} I_\lambda(\mu, x)\mu\,d\omega} \tag{3a}$$

$$m_{\lambda_-} = \frac{\int_{2\pi_-} I_\lambda(\mu, x)\,d\omega}{\int_{2\pi_-} I_\lambda(\mu, x)\mu\,d\omega} \tag{3b}$$

$$\delta_{\lambda_+} = \frac{\int_{2\pi_-} d\omega \int_{2\pi_+} I_\lambda(\mu', x)S(\mu, \mu')\,d\omega'}{4\pi \int_{2\pi_+} I_\lambda(\mu', x)\,d\omega'} \tag{4a}$$

$$\delta_{\lambda_-} = \frac{\int_{2\pi_+} d\omega \int_{2\pi_-} I_\lambda(\mu', x)S(\mu, \mu')\,d\omega'}{4\pi \int_{2\pi_-} I_\lambda(\mu', x)\,d\omega'}. \tag{4b}$$

If I_λ is isotropic within the limits of the hemispherical angle m_{λ_+} and m_{λ_-} turn out to be equal [11]

$$m_{\lambda_+} = m_{\lambda_-} = m_\lambda = 2$$

while the coefficients δ_{λ_+} and δ_{λ_-} for the axisymmetrical characteristic curves of arbitrary shape are also found to be equal to one another and are determined from the following

$$\delta_{\lambda_+} = \delta_{\lambda_-} = \delta_\lambda = \frac{1}{8\pi^2} \int_{2\pi_+} d\omega \int_{2\pi_-} S(\mu, \mu')\,d\omega'.$$

Here we shall use the coefficients m_λ, and δ_λ for the above case in order to simplify the analysis.

Solving (1a) and (1b), E_{λ_+} and E_{λ_-} come out to be:

$$E_{\lambda_+} = \frac{A}{2}\left(1 - \frac{K_\lambda}{\alpha_\lambda}\right)e^{2K_\lambda x} + \frac{B}{2}\left(1 + \frac{K_\lambda}{\alpha_\lambda}\right)e^{-2K_\lambda x} + \frac{\alpha_\lambda R_\lambda}{4K_\lambda^2} \tag{5}$$

$$E_{\lambda_-} = \frac{A}{2}\left(1 + \frac{K_\lambda}{\alpha_\lambda}\right)e^{2K_\lambda x} + \frac{B}{2}\left(\frac{K_\lambda}{\alpha_\lambda} - 1\right)e^{-2K_\lambda x} - \frac{\alpha_\lambda R_\lambda}{4K_\lambda^2} \tag{6}$$

where

$$K_\lambda^2 = \alpha_\lambda(\alpha_\lambda + 2\delta_\lambda \gamma_\lambda).$$

Boundary conditions for the layer are

$$E_{\lambda_+}\Big|_{x=x_1} = E^+_{\lambda_{x_1}} \quad \text{and} \quad E_{\lambda_-}\Big|_{x=x_2} = E^-_{\lambda_{x_2}}.$$

Substituting in (5) and (6) we get A and B

$$A = \frac{\left(1 - \frac{K_\lambda}{\alpha_\lambda}\right)e^{-2K_\lambda x_2}E^+_{\lambda_{x_1}} + 2\left(1 + \frac{K_\lambda}{\alpha_\lambda}\right)e^{-2K_\lambda x_1}E^-_{\lambda_{x_2}} - \frac{\alpha_\lambda R_\lambda}{2K_\lambda^2}\left[\left(1 - \frac{K_\lambda}{\alpha_\lambda}\right)e^{-2K_\lambda x_2} - \left(1 + \frac{K_\lambda}{\alpha_\lambda}\right)e^{-2K_\lambda x_1}\right]}{\left(1 - \frac{K_\lambda}{\alpha_\lambda}\right)^2 e^{2K_\lambda(x_1 - x_2)} - \left(1 + \frac{K_\lambda}{\alpha_\lambda}\right)^2 e^{-2K_\lambda(x_1 - x_2)}} \tag{7}$$

$$B = \frac{2E^+_{\lambda_{x_1}} - A\left(1 - \frac{K_\lambda}{\alpha_\lambda}\right)e^{2K_\lambda x_1} - \frac{\alpha_\lambda R_\lambda}{2K_\lambda^2}}{\left(1 + \frac{K_\lambda}{\alpha_\lambda}\right)e^{-2K_\lambda x_1}}. \tag{8}$$

Transmissivity of the layer from x_1 to x_2:

$$D = \frac{E_{\lambda_+}\Big|^{x=x_2}_{E^-_{\lambda_{x_2}}=0}}{E^+_{\lambda_{x_1}}} = \frac{1}{E^+_{\lambda_{x_1}}}\left[\frac{A_D}{2}\left(1 - \frac{K_\lambda}{\alpha_\lambda}\right)e^{2K_\lambda x_2} + \frac{B_D}{2}\left(1 + \frac{K_\lambda}{\alpha_\lambda}\right)e^{-2K_\lambda x_2} + \frac{\alpha_\lambda R_\lambda}{4K_\lambda^2}\right] \tag{9}$$

where

$$A_D = \frac{\left(1 - \frac{K_\lambda}{\alpha_\lambda}\right)e^{-2K_\lambda x_2}E^+_{\lambda_{x_1}} - \frac{\alpha_\lambda R_\lambda}{2K_\lambda^2}\left[\left(1 - \frac{K_\lambda}{\alpha_\lambda}\right)e^{-2K_\lambda x_2} - \left(1 + \frac{K_\lambda}{\alpha_\lambda}\right)e^{-2K_\lambda x_1}\right]}{\left(1 - \frac{K_\lambda}{\alpha_\lambda}\right)^2 e^{2K_\lambda(x_1 - x_2)} - \left(1 + \frac{K_\lambda}{\alpha_\lambda}\right)^2 e^{-2K_\lambda(x_1 - x_2)}} \tag{10}$$

and

$$B_D = \frac{2E^+_{\lambda_{x_1}} - A_D\left(1 - \frac{K_\lambda}{\alpha_\lambda}\right)e^{2K_\lambda x_1} - \frac{\alpha_\lambda R_\lambda}{2K_\lambda^2}}{\left(1 - \frac{K_\lambda}{\alpha_\lambda}\right)e^{-2K_\lambda x_1}}. \tag{11}$$

Reflectivity "R" of the layer:

$$R = \frac{E_{\lambda^-}\Big|^{x=x_1}_{E^-_{\lambda_{x_2}}=0}}{E^+_{\lambda_{x_1}}}$$

$$= \frac{1}{E^+_{\lambda_{x_1}}}\left[-\frac{A_D}{2}\left(1 + \frac{K_\lambda}{\alpha_\lambda}\right)e^{2K_\lambda x_1} - \frac{B_D}{2}\left(1 - \frac{K_\lambda}{\alpha_\lambda}\right)e^{-2K_\lambda x_1} - \frac{\alpha_\lambda R_\lambda}{4K_\lambda^2}\right]. \tag{12}$$

Absorptivity "A_{Sc}" $= 1 - D - R$

$$= 1 - \frac{1}{E^+_{\lambda_{x_1}}}\left[\frac{A_D}{2}\left\{\left(1 - \frac{K_\lambda}{\alpha_\lambda}\right)e^{2K_\lambda x_2} - \left(1 + \frac{K_\lambda}{\alpha_\lambda}\right)e^{2K_\lambda x_1}\right\} + \frac{B_D}{2}\left\{\left(1 + \frac{K_\lambda}{\alpha_\lambda}\right)e^{-2K_\lambda x_2} - \left(1 - \frac{K_\lambda}{\alpha_\lambda}\right)e^{-2K_\lambda x_1}\right\}\right]. \tag{13}$$

Equation (13) is the expression for absorptivity which includes the effect of scattering. Absorption coefficients calculated by this method are used in the calculation of the heat radiation term of the Appendix given below

$$\frac{dQ_{R_\lambda}}{d\tau_\lambda} = 2\pi \int_0^1 I^+_\lambda(0,\mu)e^{-\tau_\lambda/\mu}\,d\mu + 2\pi \int_0^1 I^-_\lambda(\tau_{0\lambda}, -\mu)e^{-(\tau_{0\lambda}-\tau_\lambda)/\mu}\,d\mu + 2\int_0^{\tau_{0\lambda}} e_{b\lambda}(t)\varepsilon_1(|\tau_\lambda - t|)\,dt - 4e_{b\lambda}(\tau_\lambda)$$

where

$$\varepsilon_n(t) = \int_0^1 \mu^{n-2} e^{-t/\mu}\,d\mu.$$

Keeping in mind that

$$\text{Absorptivity} = 1 - e^{-\alpha_\lambda(|x_2 - x_1|)}$$

$$\therefore \quad A_{Sc} = 1 - e^{-\alpha_{\lambda_{Sc}}(|x_2 - x_1|)} \tag{14}$$

$\alpha_{\lambda_{Sc}}$ thus calculated is the coefficient which takes into account the effect of scattering also.

It should be noted that the equation for $dQ_{R\lambda}/d\tau_\lambda$ above is for an absorbing and emitting medium. Substituting the coefficient $\alpha_{\lambda_{Sc}}$ calculated from (13) and (14) for the regular absorption coefficient α_λ, this equation will calculate the radiation transfer for an absorbing, emitting and scattering medium.

$\therefore$ τ_λ in the equation is

$$\tau_\lambda = \int_0^x \alpha_{\lambda_{Sc}}\,dx.$$

The Appendix gives the equations for the boundary-layer flow over a flat plate. Using the heat radiation term in these equations as described above, one can get the solution of the boundary-layer flow of an absorbing, emitting and scattering medium over a flat plate. The solution technique of Patankar and Spalding [21] is used for the boundary-layer equations. The boundary layer is divided into sublayers for the finite-difference solution of the boundary-layer equations. Each layer is considered isothermal for the calculation of its absorptivity while temperature of one layer may be different from the next. Thus in the above method in the positive x-direction $E^+_{\lambda_{x_2}}$ for the first layer is equal to $E^+_{\lambda_{x_1}}$ for the second layer and so on.

RESULTS

The numerical solution technique of Patankar and Spalding [19] was used for the boundary-layer equations given in the Appendix, which are for turbulent flow over a flat plate. The thermal radiation term in these equations was calculated by two different methods and separate subroutines were written for both methods in the computer program. In Method I, the equations given in the Appendix were used. The second computer program was written for the present method (Method II), explained in the Theoretical Development. The boundary layer was divided into sublayers. In order to find the number of sublayers needed to get a reasonably true picture of the medium, the differences in the temperatures of each layer from the preceding layer were found as percentage of the temperature of the preceding layer. It was found that in order to have the temperature differences up to and below 2%, 17 sublayers are needed. Twenty-three sublayers of the boundary layer were required in order to have the temperature difference up to and below 1.5% except at the layer closest to the wall where the difference was less than 1.6%. Also 36 sublayers of the boundary layer were required in order to have the temperature differences below 1.0% except in the layers closest to the wall. It was decided, for the purpose of this analysis, to divide the boundary layer into 30 sublayers. In this case the temperature differences between adjacent layers were below 1.15% except for the layers closest to the wall.

Figures 2 and 3 show a comparison of the results calculated by equations in the Appendix (Method I) with the results obtained by Love *et al.* [3]. This is done to establish the correctness of the radiative heat-transfer calculations by Method I, so that later the results of calculations by Method II may be compared with those of Method I with suitable boundary conditions. Figure 2 shows that for $T_\infty = 1000°R$, Method I shows QR/QW to be smaller than that calculated by Love *et al.* [3], the difference being more near the leading edge. But Figure 3 shows that Method I calculates the total heat transfer to the wall to be more than that calculated by Love *et al.* [3], the difference being more near the leading edge. This suggests that the difference is in the calculations of QW rather than QR. Analysis of Figs. 2 and 3 support this

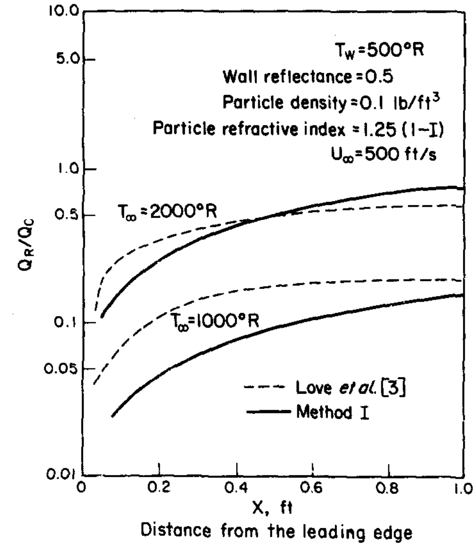

FIG. 2. Ratio of the radiative heat transfer with respect to the convective heat transfer at the surface of the plate vs distance from the leading edge of the plate.

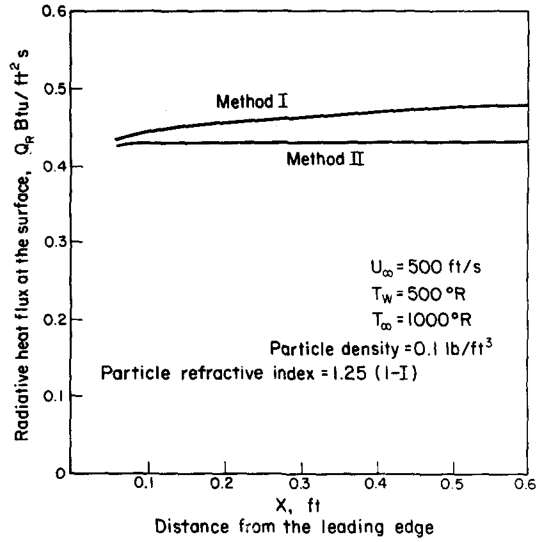

FIG. 4. Radiative heat transfer at the surface vs distance from the leading edge of the plate.

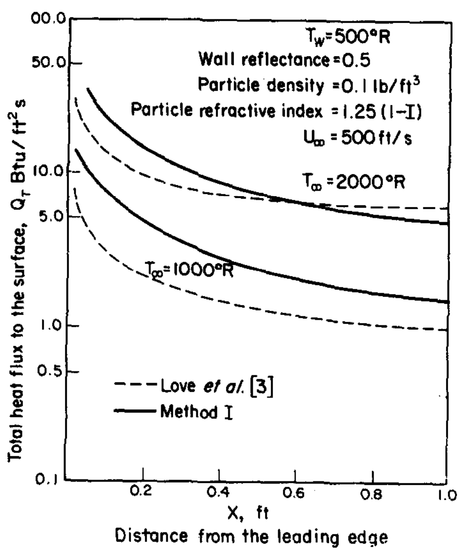

FIG. 3. Total heat transfer at the surface vs the distance from the leading edge of the flat plate.

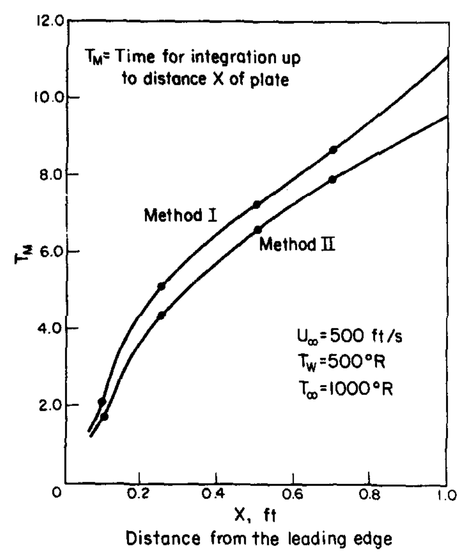

FIG. 5. Computer time used for the solution of equations vs distance from the leading edge of the flat plate.

observation. For example, at $x = 0.2$ ft the difference in QR is only 1.5% while at $x = 0.5$ ft it is 8% and at $x = 0.7$ ft it is 1.4%. However, no generalizations can be drawn from this case study.

Figure 4 shows a comparison of QR at the wall, calculated by Method I and that calculated by Method II. The comparison is quite good. Figure 5 shows that there is a considerable reduction in costly computer time by Method II. For example, if the integration of the boundary layer equations is done up to 1 ft from the leading edge, there is a saving of over 1 min and 25 s. Therefore, Method II, while being accurate, is also faster. It can be easily incorporated into the heat radiation analyses which use absorptivity or absorption coefficient and where scattering has previously been neglected because of complex analysis [17–19].

REFERENCES

1. S. Chandrasekhar, *Radiative Transfer*. Dover, New York (1960).
2. T. J. Love, An investigation of radiant heat transfer in absorbing, emitting and scattering media, Report ARL 63-3, Office of Aerospace Research, U.S. Air Force (January 1963).
3. T. J. Love, L. W. Stockham, F. C. Lee, W. A. Munter and Y. W. Tsai, Radiative heat transfer in absorbing, emitting and scattering media, Report ARL 67-210, Office of Aerospace Research, U.S. Air Force (December 1967).
4. T. J. Love and J. F. Beattie, Experimental determination of thermal radiation scattering by small particles, Report ARL 65-110, Office of Aerospace Research, U.S. Air Force (June 1965).
5. H. L. Beach, M. N. Ozisik and C. E. Siewert, Radiative transfer in linearly anisotropic scattering, conservation and non-conservative slabs with reflective boundaries, *Int. J. Heat Mass Transfer* **14**(10), 1551–1165 (1971).

6. M. A. Boles and M. A. Ozisik, Simultaneous ablation and radiation in an absorbing, emitting and isotropically scattering medium, *J. Quantve Spectrosc. Radiat. Transfer* **12**, 839–847 (1972).
7. F. Shahrokhi and P. Wolf, Numerical solution to radiative heat transfer equation for scattering medium, *AIAA Jl* **6**(9), 1748–1752 (1968).
8. C. F. Sanders and J. M. Lenoir, Radiative transfer through a cloud of absorbing, scattering particles, *A.I.Ch.E. Jl* **18**(1), 155–162 (1972).
9. A. R. Nagy and J. M. Lenoir, Absorption and scattering of thermal radiation by a cloud of small particles, *A.I.Ch.E. Jl* **16**(2), 286–292 (1970).
10. H. C. Hottel, A. F. Sarofim, I. A. Vasalos and W. H. Dalzell, Multiple scatter: comparison of theory with experiment, *J. Heat Transfer*, 285–291 (1970).
11. V. N. Adrianov, Role of scattering in radiant heat transfer, *Heat-Transfer—Soviet Res.* **1**(4), 126–132 (1969).
12. Yu A. Surinov and A. A. Kobishev, Determination of a generalised angular coefficient of radiation between bodies separated by an absorbing and scattering medium, *Heat-Transfer—Soviet Res.* **2**(4), 116–132 (1970).
13. M. G. Kaganer, Combined heat transfer by radiation and thermal conduction in absorbing and dispersing media, *Acad. Nauk. Izv. Energetika i Transport* No. 6, 94–99 (1968).
14. V. N. Andrianov, Differential methods of radiative heat transfer calculations, *Heat Transfer—Soviet Res.* **1**(4), 111–125 (1969).
15. A. B. Blokh, Thermal radiation in disperse media, *Proceedings of 2nd All Soviet Conference on Heat and Mass Transfer, May* 1964, *Minsk, Soviet Union*, Vol. 5, pp. 326–338 (1967).
16. E. M. Sparrow and R. D. Cess, *Radiation Heat Transfer*, Brooks/Cole, Belmont, Ca (1966).
17. J. M. Elliott, R. I. Vachon, D. F. Dyer and J. R. Dunn, Application of the Patankar/Spalding finite difference procedure to turbulent radiating boundary layer flow, *Int. J. Heat Mass Transfer* **14**(5), 667–672 (1971).
18. J. H. Chin, Radiation transport for stagnation flows including effect of lines and ablation layer, AIAA Paper No. 68–664 (June 1968).
19. S. Kamalam and S. S. R. Murty, Boundary layer characteristics with radiant energy transfer under adverse pressure gradient, AIAA Paper No. 73–117 (January 1973).
20. C. M. Chu, G. C. Clark and S. W. Churchill, Tables of angular distribution coefficients for light scattering by spheres, University of Michigan Press, An Arbor, Michigan (1957).
21. S. V. Patankar and D. B. Spading, *Heat and Mass Transfer in Boundary Layers*. Morgan–Grampian, London (1967).
22. F. C. Chromey, Evaluation of Mie equations for coloured spheres, *J. Opt. Soc. Am.* **50**(7), 730–737 (1960).

APPENDIX

The equations for turbulent boundary layer are:

Continuity:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial Y}(\rho v) = 0 \tag{A.1}$$

x-momentum:

$$\rho u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial Y} = \frac{\partial}{\partial Y}\left(\mu_{\text{eff}} \frac{\partial u}{\partial Y}\right) - \frac{dP}{dx}\left(\frac{dP}{dx} = 0 \text{ for flat plate}\right) \tag{A.2}$$

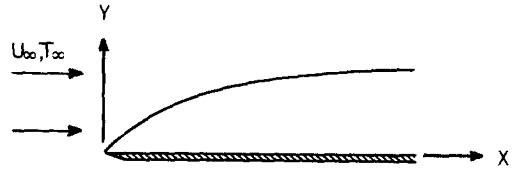

FIG. A1.

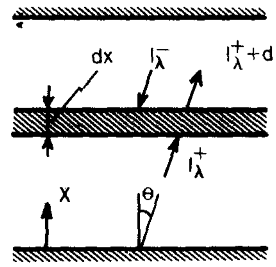

FIG. A2.

Energy:

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial Y} = \frac{\partial}{\partial Y}\left\{\frac{\mu_{\text{eff}}}{Pr_{\text{eff}}}\frac{\partial H}{\partial Y} + \mu_{\text{eff}}\left(1 - \frac{1}{Pr_{\text{eff}}}\right)\frac{1}{2}\frac{\partial u^2}{\partial Y}\right\} - \operatorname{Div} Q_R \tag{A.3}$$

where u, p and H are the time averaged values of the fluctuating turbulent quantities. μ_{eff} and Pr_{eff} are called "effective" transport properties and take into account the turbulent and the laminar contributions to shear and heat-flux respectively. These are solved by Patankar and Spading technique with the source-term for the energy equation for one dimensional heat transfer for an absorbing and emiting medium given by

$$\operatorname{Div} Q_R = \int_0^{\lambda} \frac{\partial}{\partial Y}(Q_{R\lambda})\,d\lambda. \tag{A.4}$$

Q_{R_λ}, as given by Sparrow and Cess [14] for an absorbing, emitting and scattering medium is

$$\begin{aligned} Q_{R_\lambda}(\tau_\lambda) = {} & 2\pi \int_0^1 I_\lambda^+(0,\mu)\, e^{-\tau_\lambda/\mu}\mu\, d\mu \\ & - 2\pi \int_0^1 I_\lambda^-(\tau_{0_\lambda}, -\mu)\, e^{-(\tau_{0_\lambda}-\tau_\lambda)/\mu}\mu\, d\mu \\ & + 2\int_0^{\tau_\lambda}\left[\frac{\alpha_\lambda}{\beta_\lambda} e_{b_\lambda}(t) + \frac{\gamma_\lambda}{4\beta_\lambda} G_\lambda(t)\right]\varepsilon_2(\tau_\lambda - t)\, dt \\ & - 2\int_{\tau_\lambda}^{\tau_{0\lambda}}\left[\frac{\alpha_\lambda}{\beta_\lambda} e_{b_\lambda}(t) + \frac{\gamma_\lambda}{4\beta_\lambda} G_\lambda(t)\right]\varepsilon_2(t - \tau_\lambda)\, dt \end{aligned} \tag{A.5}$$

and

$$\begin{aligned} \frac{dQ_{R_\lambda}}{d\tau_\lambda} = {} & 2\pi \int_0^1 I_\lambda^+(0,\mu)\, e^{-\tau_\lambda/\mu}\, d\mu \\ & + 2\pi \int_0^1 I_\lambda^-(\tau_{0_\lambda}, -\mu)\, e^{-(\tau_{0\lambda}-\tau_\lambda)/\mu}\, d\mu \\ & + 2\int_0^{\tau_{0\lambda}}\left[\frac{\alpha_\lambda}{\beta_\lambda} e_{b\lambda}(t) + \frac{\gamma_\lambda}{4\beta_\lambda} G_\lambda(t)\right]\varepsilon_1(|\tau_\lambda - t|)\, dt \\ & - 4\left(\frac{\alpha_\lambda}{\beta_\lambda} e_{b_\lambda}(\tau_\lambda) + \frac{\gamma_\lambda}{4\beta_\lambda} G_\lambda(\tau_\lambda)\right) \end{aligned} \tag{A.6}$$

where

$$\mu = \cos\theta, \quad \tau_\lambda = \int_0^x \beta_\lambda\, dx$$

$$G_\lambda = \int_0^{2\pi}\int_{-1}^{1} I_\lambda(\mu', \phi') S(\mu, \phi; \mu', \phi')\, d\mu'\, d\phi'$$

$S(\mu, \phi; \mu', \phi')$ is the scattering function. The scattering function was obtained from Love [2] for Mie scattering, where it is represented as

$$S(\mu, \mu') = 1 + \sum_{n=1}^{n=\infty} a_n P_n(\mu, \mu')$$

where Pn are the Legendre Polynomials and a_n are the coefficients listed in the tables of Chu *et al.* [20] for different values of the particle size parameter "Z" and refractive index. Scattering function represented this way is the axially symmetric scattering function. Love and Beattie [4] have presented some measurements and have shown that the assumption of axially symmetric function is reasonable for random orientation of the particles.

The extinction and scattering functions are taken from Chromey [22] for values of Z and complex refractive index. The absorption coefficients for the host gases are taken from Elliot [17], who has represented them as three band approximation.

ANALYSE DU TRANSFERT THERMIQUE PAR RAYONNEMENT EN UTILISANT UNE ABSORPTIVITE EFFECTIVE POUR L'ABSORPTION, L'EMISSION ET LA DIFFUSION

Résumé—On utilise dans cette étude une approche non conventionnelle pour résoudre les équations du transfert thermique par rayonnement, à travers un milieu absorbant, émetteur et diffusant. On développe une expression de l'absorptivité effective d'une couche de ce milieu, en incluant l'effet de l'absorption, de l'émission et de la diffusion. Cette nouvelle "absorptivité effective en présence de diffusion" peut être utilisée pour évaluer le transfert thermique par rayonnement entre deux surfaces avec un milieu immobile ou en écoulement. Dans cette étude, cette expression permet l'évaluation du transfert dans une couche limite turbulente sur plaque plane. Le transfert thermique par rayonnement calculé par la méthode décrite est comparé avec celui calculé par les méthodes conventionnelles. On montre que la méthode développée ici est aussi précise mais plus rapide que les techniques actuelles et qu'elle économise un temps considérable de calcul sur ordinateur.

UNTERSUCHUNG DES STRAHLUNGSWÄRMEAUSTAUSCHES MIT HILFE EINER EFFEKTIVEN ABSORPTIONSZAHL FÜR ABSORPTION, EMISSION UND STREUUNG

Zusammenfassung—Zur Lösung der Strahlungs-Wärmeaustausch-Gleichungen bei absorbierenden, emittierenden und streuenden Medien wird ein unkonventioneller Ansatz verwendet. Dabei wird eine effektive Absorptionszahl eingeführt, welche Absorption, Emission und Streuung beinhaltet. Diese neue "effektive Absorptionszahl mit Berücksichtigung der Streuung" kann dazu verwendet werden, den Strahlungswärmeaustausch zwischen zwei Flächen, zwischen denen sich ein strömendes oder ruhendes Medium befindet, zu berechnen. Der Ausdruck wird in dieser Arbeit dazu benutzt, den Strahlungswärmeaustausch in turbulenten Grenzschichtströmungen über einer ebenen Platte zu ermitteln. Die auf diese Weise ermittelten Ergebnisse werden mit den mit konventionellen Methoden berechneten Werten verglichen. Die hier vorgeschlagene Methode ist ebenso genau und führt schneller zum Ziel, so daß beträchtliche Einsparungen an Rechenzeit ermöglicht werden.

АНАЛИЗ ЛУЧИСТОГО ТЕПЛОПЕРЕНОСА С ПОМОЩЬЮ ЭФФЕКТИВНОГО КОЭФФИЦИЕНТА ПОГЛОЩЕНИЯ ДЛЯ ПОГЛОЩАЮЩИХ, ИЗЛУЧАЮЩИХ И РАССЕИВАЮЩИХ СРЕД

Аннотация — Для решения уравнений лучистого переноса тепла через поглощающие, излучающие и рассеивающие среды используется нестандартный подход. Выведено выражение для эффективного коэффициента поглощения слоя среды, учитывающее эффекты поглощения, излучения и рассеивания. Этот новый «эффективный коэффициент поглощения при наличии рассеивания» может быть использован для расчета лучистого теплопереноса между двумя поверхностями при наличии между ними текучей или покоящейся среды. В данном исследовании полученное выражение используется для оценки лучистого теплообмена в турбулентном пограничном слое на плоской пластине. Результаты по лучистому переносу тепла, полученные с помощью этого метода, сравниваются с данными, полученными по общепринятой методике. Видно, что по сравнению с существующими методами разработанный метод является точным и позволяет ускорить расчеты на ЭВМ.